A necessary condition for partially funded pension systems

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Introduction

The work finds classification in the framework of “logically sustainable pension systems”, which are pension systems whose sustainability is founded on logical-mathematical rules rather than actuarial forecasting.

- Angrisani, M. 2006. Funded and unfunded systems: two ends of the same stick. *Paper presented at the 28th International Congress of Actuaries.* 28 May - 2 June, 2006. Paris, France.
- Angrisani, M. 2008. The logical sustainability of the pension system. *Pure Mathemathics and Applications* 19 (1): 67-81

The Model Functions

For each t in T, we have that:

$\alpha(t)$ is the contribution rate, with $\alpha(t) \geq 0$

$C(t) \geq 0$ and $W(t)$ are the instantaneous flow of contributions and the instantaneous flow of wages, respectively, with $C(t) \geq 0$, $W(t) > 0$, and $C(t) = \alpha(t) W(t)$

$P(t)$ is the instantaneous flow of the pension expenditure, with $P(t) > 0$

$F(t)$ is the pension system fund

$r(t)$ is the instantaneous rate of return on fund

$L^T(t)$ is the total pension liability, with $L^T(t) > 0$

$r_L(t)$ is the instantaneous rate of return on the total pension liability.

The Model Definitions

Definition 1. *A pension system is sustainable in time interval T if and only if* $F(t) \geq 0$ *for each t in T.*

Definition 2. *For each instant t in T, the unfunded pension liability is*

$$L^{UN}(t) = L^{T}(t) - F(t).$$

$$L^{T}(t) = L^{UN}(t) + F(t)$$

It is assumed that $L^{T}(t) \geq F(t)$ for all values t in T. The unfunded pension liability is then subjected to the condition $L^{UN}(t) \geq 0$, for all values t in T.

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The Basic Equations of the Model

Equation 1. $\dot{F}(t) = F(t)r(t) + \alpha(t)W(t) - P(t)$

Equation 2. $\dot{L}^T(t) = L^T(t)r_L(t) + \alpha(t)W(t) - P(t)$

Equation 2. uses two important control variables of the pension system, the rate of return on the pension liability, $r_L(t)$, and the contribution rate, $\alpha(t)$.

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The Model Indicators (1)

1. Function $\nu(t)$ is the divisor of the total pension liability in the pension liability to retirees at time *t*, i.e.,

$$\nu(t)=\frac{L^T(t)}{L^P(t)}, \text{ with } \nu(t)\geq 1.$$

2. Function $\gamma(t)$ is the divisor of the pension liability of retirees in the pension expenditure at time t, i.e.,

$$\gamma(t)=\frac{L^P(t)}{P(t)}.$$

3. Function $\gamma(t)\nu(t)$ is the divisor of the total pension liability in the pension expenditure at time t, i.e.,

$$\gamma(t)\nu(t)=\frac{L^T(t)}{P(t)}.$$

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The Model Indicators (2)

4. Function $\beta(t)$ is the level of the unfunded pension liability respect to wages at time t, i.e.,

$$\beta(t) = \frac{L^{UN}(t)}{W(t)}.$$

5. $D_c(t) = \frac{F(t)}{L^T(t)}$ is the *degree of funding of the pension liability*

6. Function $\alpha^{UN}(t)$ is the level of the unfunded contribution rate at time t, i.e.,

$$\alpha^{UN}(t) = \frac{\beta(t)}{\gamma(t)\nu(t)},$$

namely is the level of the contribution rate necessary to cover the unfunded pension expenditure.